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Sensory and Cognitive Factors in the Processing of Visual Velocity

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A symmetrical 6 × 6 factorial design of distances and durations served to produce either 36 different moving stimuli (real movement condition) or 36 static displays separately containing the respective stimulus components (cognitive movement condition). Different metric rules underlay the two types of velocity judgments: Perceptual estimations of real movement obeyed a ratio model, whereas conscious estimations of implied movement obeyed an additive model. Valuation operations differed, too; the scales underlying real velocity were nonlinearly related to the even more compressive scales that underlay cognitive velocity. Implications of these results for velocity research are discussed.

Specifying the invariant physical correlates of motion perception has been an extremely difficult problem in the field of visual velocity research. The crux of the problem lies in the fact that an object can be made to appear to move by a relatively large variety of physical arrangements, but only a single class of these—surprisingly, perhaps, one of the least explored—is real movement, where stimuli undergo actual and continuous spatial displacement. Our most recent textbooks still devote a markedly larger space to the treatment of the different illusions of movement than to the perception of really moving targets. In fact, beyond the usual documentation of the survival value of a veridical movement perception and some discussion of threshold (e.g., J. F. Brown, 1931b; R. H. Brown, 1955; Cohen & Bonnet, 1972), of background and context (e.g., J. F. Brown, 1931a, 1931c), and of detector mechanisms (e.g., Gregory, 1966), virtually nothing is extant on sensory registration of suprathreshold movement nor on the psychophysics of velocity. Why?

A Paradox of Visual Velocity

We believe the psychophysical analysis of visual motion has been beset by a fundamental conceptual difficulty that can be justifiably called *the paradox of psychological velocity.* The problem, recurrently appearing in virtually all studies of real movement, can be stated in very simple terms. In physics, velocity is given by the ratio of the spatial and temporal extents traversed by a body in motion. This formal definition provides a natural context for an analogous dividing rule of psychological integration. Given the appealing simplicity of this model, it has been invariably (though, at times, implicitly) assumed to underlie judgments of velocity (e.g., J. F. Brown 1931c; Dember & Warm, 1979; Drosler, 1978; Mashhour, 1964; Rachlin, 1966; Wilkening, 1981). Yet dividing judged distance by judged duration of a moving stimulus does not give its judged velocity. Despite occasional acknowledgment of the complexities posed by this lack of correspondence between the assumed metric structure and the overt velocity response (e.g., Dember & Warm, 1979, p. 315), the puzzle has rarely been given explicit attention. When it has been examined, even elementary treatments have resulted almost instantaneously in either rejection of the psychophysical power law (Drosler, 1978) or a modification of the simple ratio model (Rachlin, 1966).

Attempts at specifying the correct metric structure underlying judgments of velocity make apparent the fact that representations of velocity rest on a complex dynamic of the stimuli involved. Fruitful analysis of psychological velocity considered as a perceptual system (cf. Gibson, 1966) is contingent, however, on the availability of appropriate multicomponential experimental techniques. Unfortunately, in this regard, psychological evidence pertinent to this issue has been obtained almost exclusively by unifactor stimulus designs in conjunction with direct scaling techniques for obtaining the response (see Algom & Cohen-Raz, 1984, for a recent review of this literature). As is well recognized now, the validity of such techniques relies on the questionable assumption that putative numerical ratios reflect actual ratios of subjective scale values (e.g., Algom & Cohen-Raz, 1984; Algom & Marks, 1984; Anderson, 1970, 1974, 1981, 1982; Curtis, Attneave, & Harrington, 1968; Garner, 1954; Torgerson, 1961). More important for the present purposes than even the fundamental validation problem is the inherent failure of unifactorial studies to provide a test of any theory of the way(s) in which distance and duration information combine in perception to produce subjective velocity.

Functional Measurement of Visual Velocity

In contradistinction, the approach advocated over the past several years by Anderson (e.g., 1970, 1974, 1981, 1982) places central emphasis on stimulus integration and on the specific form of concatenation of different subjective components into a unitary overt response. The method makes use of judgments of multicomponential stimuli (not necessarily *perceived* as such) in conjunction with an algebraic model for combining the components' scale values, as the basis for scaling psychological variables such as velocity. If a multiplicative (or ratio) structure

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is directly evident in the numerical judgments, then the responses (such as the marginal means in a factorial design) are linearly related to the underlying scale values. This solution to the scaling problem is designated *functional measurement*.

Recently, we made a first attempt at a systematic application of functional measurement methodology to study velocity interactions (Algom & Cohen-Raz, 1984). In particular, we assessed how distance and duration are integrated in judgments of the velocities of moving stimuli, produced by a complete factorial design of distances and durations. Our results were consistent with a dividing model of stimulus integration. Beyond the powerful establishment of the ratio metric for psychological velocity, this study enabled us to offer a clue for a possible resolution of the velocity puzzle. Psychophysical analyses revealed that the values of distance and duration conforming to the dividing model were not *the* psychological values of distance and duration. Rather, the uncovered ratio structure operated on a set of implicit scale values of distance and duration that were nonlinearly related to the psychological values resulting from explicit judgments of distance and duration.

Real Versus Cognitive Movement

In the present study we contrast some of the psychological processes that occur when people judge the velocity of moving targets with the processes that underlie judgments of velocity extracted from stationary environments (i.e., that do not derive from real movement). In particular, we seek to compare integration rules and scale values for velocity under two rather different conditions. In one condition, the real-movement condition (Experiment 1), the combination of stimulus components (i.e., distance and duration) is fairly automatic, falling under the operation of the perceptual system. In the other condition, designated cognitive velocity (Experiment 2), the necessary distance and duration information was presented separately (rather than intermixed into a single stimulus event as in real motion) in a static display. Neither physical movement nor a psychological illusion of movement was produced in that condition. Judgments of velocity were induced instead in a wholly conscious way under the experimenter's direction. In this condition, the combination of components is, perforce, discretionary, coming under cognitive control. Our goal was to see how velocity is processed under adequate (real velocity) and under inadequate (cognitive velocity) stimulus conditions (cf. Boring, 1942). As mentioned earlier, Algom and Cohen-Raz (1984) showed that perception of real velocity is a dividing process resting on a unique set of internal scale values for distance and duration. How do rules of integration and underlying scale values compare when subjects are asked to judge velocity based on the same stimulus parameters presented not naturally as interlocked constituents of motion but separately as two distinct (and static) events? Comparing the results obtained under the two experimental conditions can help to disentangle purely sensory or perceptual processes (presumably operating in real movement) from cognitive mechanisms and transformations that presumably operate in the processing of cognitive movement.

Experiment 1: Real Velocity

In the present experiment we followed the conditions of our earlier experiment (Algom & Cohen-Raz, 1984) and used a relatively constrained factorial design of distances and durations and a more restricted stimulus range. Nevertheless, the range of values used was sufficiently large to avoid too strong an influence of some well documented systematic effects in the derivation of the psychophysical function (e.g., Poulton, 1968; Teghtsoonian, 1971, 1973). The main burden, though, of the present experiments is metric structure, rather than a detailed description of the psychophysical function. (For a precise parametric determination of the latter, see Algom & Cohen-Raz, 1984). Beyond the valuable replication, these data serve as the standard perceptual model for visual velocity to be compared with models of subjective velocity derived under different—discontinuous—stimulus conditions.

Method

Subjects. Eight undergraduate students at Bar-Ilan University participated in this experiment. Their ages ranged from 21 to 27 years, and none of them was familiar with the theoretical issues being investigated nor with the method of magnitude estimation.

Apparatus. Stimulus production was the same as in the previous study (Algom & Cohen-Raz, 1984). A luminous spot (diameter 0.8 cm) moving at a constant velocity for some fixed distance and duration in a linear track appeared on a cathode-ray tube (CRT) display (Textronics 422) from which it was transmitted and displayed on a larger TV screen (PYE 26"). The spot had a value of 210 lux, and the background had a value of approximately 150 lux (Lamda LI-170); measurements were made on the surface of the screen. A small digital computer (PDP-11) was used to produce and administer the moving stimuli.

The subject sat in a dimly lighted room (2.8 m × 1.4 m) in which the only anchoring was the frame of the screen (70.5 cm × 47 cm) showing the luminous dots. The displays were viewed binocularly at a distance of approximately 1 m. Although head and body movements were not restricted, the subjects were instructed to rest their heads on a chinrest and to fixate on the center of the display.

Procedure. The experimental design was that used by Algom and Cohen-Raz (1984). Six different levels of distance (d_1–d_6 = 20, 30, 40, 60, 80, and 160 mm) were combined factorially with 6 levels of duration (t_1–t_6 = 0.5, 1, 1.5, 2, 3, and 4 s), making 36 different velocity stimuli in all. (A certain degree of redundancy is evident, though, because different distances and durations may yield constant values of velocity.) Stimuli were presented one at a time to the subject for judgment. Each subject received two replicates of the stimulus matrix in the first session and three replicates in a second session, thus giving five judgments per stimulus in all. In addition, all subjects participated in a preliminary practice session to become familiarized with both stimulus setting and the method of judgment. The order of presentation of the stimuli was irregular and was different for each subject. (Orderly sequences were intentionally avoided.)

The method of response was magnitude estimation. Subjects were instructed to assign to the first stimulus whatever number seemed most appropriate to represent its velocity; then they were instructed to assign numbers, in proportion, to other stimuli. If no movement was seen, the subjects were to assign the number zero (which none of them did). Subjects were told that they could use whole numbers, decimals, and fractions as needed.

Results and Discussion

Pooled data. The magnitude estimates of subjective velocity given to each stimulus were averaged geometrically, and these means are plotted in Figure 1 as a function of the log of the distance traveled by the target. The parameter is duration: Each contour represents motion exposed for a different constant duration. Because the velocity estimates are plotted on a linear scale, the hypothesis of a dividing model operating on the psychological representations of distance and duration—transforming them onto subjective velocity—may be assessed by visual inspection. Perhaps the most striking feature of this family of functions is their tendency to diverge at the upper right. The marked divergence, as well as the overall linear fan shape of the data, indicates a Velocity = Distance ÷ Duration rule of stimulus integration. Though there appear to be some departures from exact bilinearity, the data seem to obey the implications of the dividing model fairly well. As a first approximation, then, these results suggest that subjects did use a simple ratio estimation process when they judged velocity.[1]

Analysis of variance confirms the visual appearance of the linear fan pattern. The overall Distance × Duration interaction is highly significant, $F(25, 175) = 3.03$, $p < .01$. Moreover, a major part of this interaction (88%) appears in the bilinear component that expresses the portion of the variance found in the ratio (or the product) of the responses to the component physical variables involved in the production of real movement. The bilinear component is highly significant, $F(1, 25) = 22.00$, $p < .01$, whereas the remainder of the interaction is not ($F < 1$).

The finding of a dividing model operating in the integration of distance and duration information onto subjective velocity replicates the outcome of our previous study (Algom & Cohen-Raz, 1984). Despite the use of a relatively constrained factorial design, a linear (rather than circular) track, and substantially smaller ranges of stimulus values than in our previous investigation, the dividing bilinear fan pattern clearly reappeared. It appears, therefore, that the normative Velocity = Distance ÷ Duration integration rule is fairly robust in adults' perception of moving objects.

As Anderson (1970, 1974, 1982) has pointed out, given a factorial design of the type used in this study and results consistent with bilinearity in the response domain, the marginal means

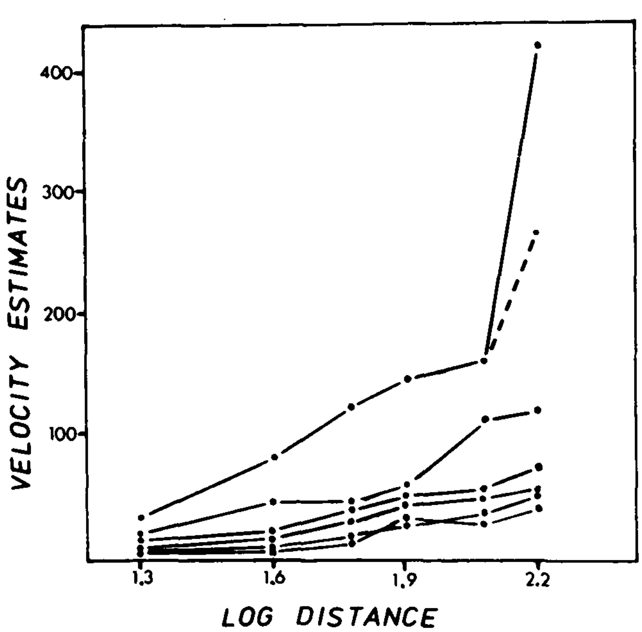

Figure 1. Magnitude estimates of velocity as a function of distance. (The parameter is duration.)

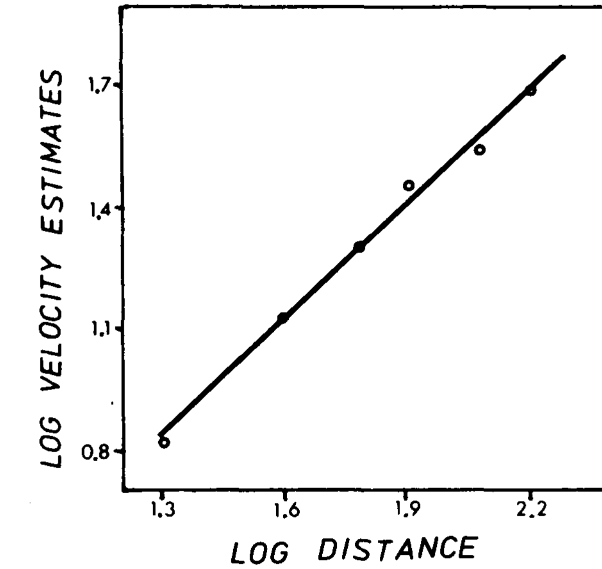

Figure 2. Marginal means of velocity judgments as a function of distance.

provide estimates of the scale values.[2] Figures 2 and 3 give these calculated scale values of subjective velocity as a function of different distances and different durations, respectively.

The fits to the power functions (straight lines in the double logarithmic coordinates) are excellent ($r^2 = .994$ for the distance marginal function and .998 for the duration marginal function). These functions were corrected here with the iterative techniques detailed in Mashhour (1964, Chapter 3). The fits to the corrected power functions are, again, excellent (r^2 equals .995 and .997). The slopes are 0.811 and 0.883, respectively.

[1] The high value at the upper right in Figure 1 comes from extreme judgments of this stimulus by one of the subjects. Excluding the data of this subject from the computations does not alter the overall results appreciably, though they come to follow more closely the bilinear fan pattern. The broken line in Figure 1 shows the value of the fastest stimulus based on the data of 7 subjects (with the data of the deviant subject excluded). A Distance × Duration analysis of variance performed on the data of the 7 subjects replicated essentially the original results, yielding a highly significant interaction term, $F(25, 100) = 2.13$, $p < .01$.

[2] The reader may have noticed some deviations from the usual procedures of functional measurement. These apply to (a) the use of magnitude estimates rather than ratings, (b) the use of geometric rather than arithmetic means, and (c) the spacing of the abscissa in Figure 1 (logarithmically rather than by marginal means). Our practice has shown that at least for the types of tasks used within the framework of the present investigation, an unbounded response scale (like magnitude estimation) is preferable to a bounded one. Given the use of magnitude estimates, the geometric mean is the proper summary statistic. Finally, spacing of the abscissa touches on complex questions relating to the choice of coefficients in the method orthogonal polynomials. Logarithmic spacing of the stimulus scale usually reflects accurately the internal scale values. In any case, *linear* spacing of the ordinate is essential for the correct visual assessment of the integration rule. For procedures similar to the ones employed here, the interested reader should refer to Algom and Cohen-Raz (1984), Algom and Marks (1984), or Marks (1979b).

The respective (corrected) power function exponents for distance and duration in our previous study were 0.62 and 0.64. Despite the appreciable difference in the absolute values (likely to stem from differences in stimulus range), both sets of exponents reveal a clearly compressive psychophysical relation for subjective velocity as a function of either distance or (the reciprocal of) duration. Moreover, the psychophysical power function relating perceived velocity to the duration of movement appears slightly, though consistently, steeper than the parallel function relating subjective velocity to the distance traversed in both investigations. Note that this exponent relation does not depend on stimulus range: Equal ranges for distance and duration were used in the present study, and the range of durations exceeded that of distances in our previous study (Algom & Cohen-Raz, 1984; see also Teghtsoonian, 1971, 1973).

Another, perhaps more complete, appraisal of the present data taken as a whole is obtained by plotting mean magnitude estimates against physical velocity for all 36 stimuli presented. Figure 4 is a threshold corrected log-log plot of these velocity judgments. The slope of this psychophysical function for velocity is 0.847, and the power fit is, again, excellent (r^2 = .978). Although the rather restricted range of velocities used does have an effect on exponent magnitude, the present value is compatible with the comparable exponent in the order of 0.634 derived in our previous study.[3]

Individual data. Two aspects of each subject's results are of special interest: (a) the psychophysical scales and (b) the algebraic rule used for concatenating the values of these scales. First, consider the underlying metric structures. The question of the particular cognitive algebra used may yield to an analysis of the bilinearity evident within each subject's entire response matrix. The pooled data (Figure 1) conform well to a ratio model. So, too, do virtually all of the individual data sets. Individual factorial plots of each subject's velocity contours were remarkably consistent in demonstrating the linear fan pattern and thus yielded additional support for an underlying dividing structure.

Analyses of variance confirmed that the data for 7 of the 8 subjects agreed with a simple dividing model. The data sets of these subjects had significant interaction terms, with a sizable and significant part of the interaction residing in the Linear × Linear component in each case. These characteristics imply a dividing rule of integration for the numerical responses. See Table 1 for details of these individual analyses. On the assumption that bilinearity exists, we can derive scale values from each subject's data matrix by calculating marginal means down columns and across rows, adjusting scale values by estimating the "true" zero points of the scales for each subject. Columns 2–3, 4–5, and 6–7 in Table 2 show, respectively, for velocity, for distance, and for duration as independent variables, the exponents of (the

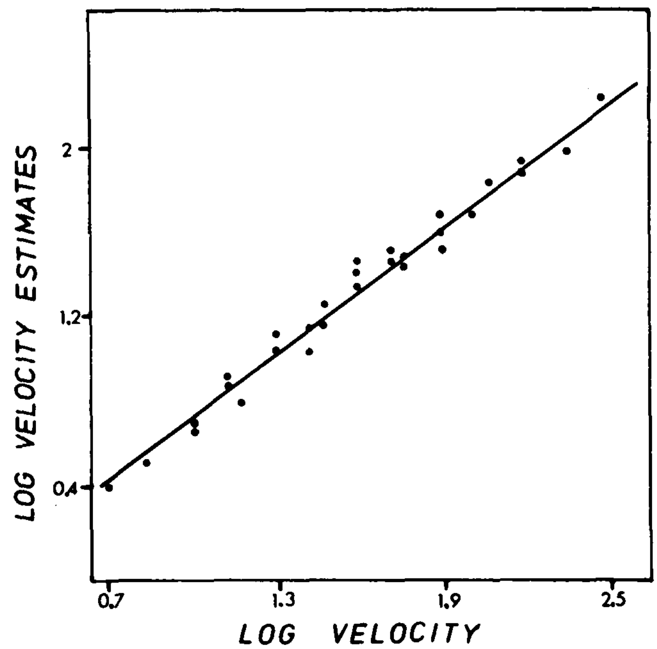

Figure 4. Psychophysical function for subjective velocity: Magnitude estimates plotted against physical velocity.

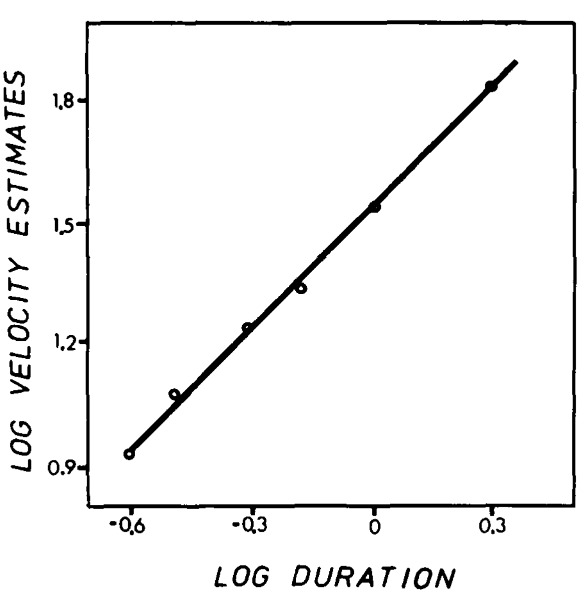

Figure 3. Marginal means of velocity judgments as a function of duration.

[3] Several studies (e.g., Mashhour, 1964; Rachlin, 1966), comparable in stimulus range to the present one, have also reported smaller than unity power function exponents for visual velocity. The relatively wide spread of exponents found in the literature for visual velocity (from below to well above unity) is probably attributable to a substantial range effect that emerges when we make comparisons across the different experiments (Algom & Cohen-Raz, 1984; Teghtsoonian, 1971, 1973). Yet even that part of previous work that has also claimed to find a compressive power relation between subjective and actual velocity is suspect because it rests on the unjustified assumption that the overt (usually numerical) response was at least an interval scale of perceived velocity (e.g., Anderson, 1972, 1981, 1982; Curtis et al., 1968; Torgerson, 1961). By contrast, the present experiment as well as our previous study both show well-defined metric structures (based on a simple dividing rule) that provide the needed validational base for deriving the scale values. Our results suggest an underlying representation for subjective velocity that is a compressive power transform of actual velocity or either of its component constituents.

Table 1
Individual Analyses of Interaction Variance for 8 Subjects

Subject	F^a	% SS[b]	F^c	F^d
1	3.13**	35	8.70**	2.04*
2	1.97**	26	6.60*	2.30**
3	13.00**	73	18.29**	0.84
4	1.22			
5	4.99**	63	15.87**	1.14
6	5.64**	71	17.81**	0.90
7	2.67**	37	9.36**	1.95*
8	1.81*	37	9.33**	1.96*

[a] Interaction. [b] Sum of squares (SS) of interaction concentrated in bilinear component. [c] Linear × Linear component. [d] Residual.
* $p < .05$. ** $p < .01$.

corrected) power functions fitted to the magnitude estimates and to the marginal means.

The fits to the power functions are good. Although there are notable interindividual differences, the present spread of exponent—somewhat less than 2:1—is, nevertheless, a bit more dense than is usually the case with magnitude estimates (e.g., J. C. Stevens & Guirao, 1964; Ramsay, 1979; see also Marks, 1974). Still, the most striking feature of the individual data is that subjective velocity grows as a negatively accelerated function of physical velocity. Individual velocity functions are distinctively compressive regardless of whether physical velocity or either of its definitional components is taken as the independent variable. Another implication that is well supported in the individual data is the faster growth of subjective velocity as a function of (the reciprocal of) duration than as a function of distance. For 7 of the 8 subjects, the exponent of the velocity psychophysical function for duration is greater than the exponent of the parallel function for distance.

Taken as a whole, the individual results substantiate the finding from the pooled data of a ratio rule of velocity integration and of an underlying compressive (power-transform) representation for subjective velocity. Although sensory representations of velocity vary somewhat from person to person with normal vision, the typical velocity scale is clearly compressive, with a representative exponent of about 0.80.

Experiment 2: Cognitive Velocity

Recall the principal aim of this study: It seeks to compare people's velocity judgments of moving targets (real velocity) with their velocity judgments based on separately presented distance and duration information (cognitive velocity). The latter judgments, it should be recognized, are constructed wholly consciously as a result of appropriate experimental instructions. The questions posed by the present experiment are as follows: Does the dividing rule for perceptual integration found for real velocity apply when the velocity estimation task comes under cognitive control? How do psychophysical measures for cognitive velocity compare with the corresponding real-velocity estimates? Finally, do exponent values for velocity as a function of distance and as a function of duration preserve their order in the cognitive velocity task?

Specification of the underlying metric structure for inferred velocity is interesting in several respects. To begin with, an immediate, popular, imaginary-movement hypothesis assumes that judgments of velocity of such an imaginary movement should follow the same dividing rule as do judgments of real movement. An across-tasks invariant dividing integration rule is also the prediction of the cognitive development approach advocated by Piaget (e.g., 1969, 1970a, 1970b; Wilkening, 1981) and sometimes labeled the logical–operational approach (e.g., Anderson & Cuneo, 1978a, 1978b; Wilkening, 1979). Veridical judgments of velocity are the result of grasping the correct formal relations between velocity, time, and distance. Given the salient presentation of both stimulus dimensions in the cognitive velocity condition and given adults' assumed mastery of the correct logical operations needed for a quantitative velocity judgment, a dividing rule of integration comes out as the natural prediction of the Piagetian approach for this condition.

Marks (1979a) has investigated characteristics of the sensory phenomenon of binaural summation. In particular, he sought to uncover the rule by which concurrent auditory stimulation is integrated across the two ears—even though the individual left-ear and right-ear components are not available to perception. He made a comparison with what he called a *cognitive summation condition* where the individual components, separated in time, were available to perception and hence came under conscious control. He found the same metric rule—although different psychophysical loudness functions—to underlie the two phenomena. The present research captures the spirit of Marks's study in both experimental strategy and theoretical direction, albeit in a completely different perceptual system.

The experimental condition that is here termed *cognitive motion* rests on a unique set of hitherto unresearched stimulus properties. Uncovering the underlying metric structure, therefore, has considerable intrinsic interest beyond reflecting on the fundamental issue of single versus multiple metric structures governing parallel sensory and cognitive phenomena. In addition, despite the above arguments and related evidence, the assumption that consciously computed velocity will follow the same rule as perceived velocity can easily be questioned on both epistemological and logical grounds. Therefore, in its broadest sense, the question posed by the present study is: Under what circumstances will a given perceptual integration rule preserve

Table 2
Power Function Exponents (Slopes) and r^2s Derived From the Marginal Means of the Factorial Designs for 8 Subjects

	Velocity		Distance		Duration	
Subject	Slope	r^2	Slope	r^2	Slope	r^2
1	0.807	.834	0.758	.980	0.872	.967
2	0.527	.930	0.658	.988	0.873	.956
3	0.978	.924	0.957	.974	0.994	.977
4	0.695	.958	0.649	.993	0.816	.993
5	0.637	.895	0.677	.960	0.819	.977
6	1.035	.853	1.011	.974	1.054	.977
7	0.748	.900	0.801	.955	0.874	.993
8	0.645	.845	0.892	.962	0.805	.991
M	0.759		0.800		0.888	

its form when the original (usually automatically performed) task comes under cognitive control?

Method

Subjects. Seven undergraduate students at Bar-Ilan University served as subjects. Two of them had served in Experiment 1. Their ages ranged from 20 to 29 years.

Apparatus. Distances and durations were generated in a manner identical to that of Experiment 1, except for the absence of movement. Stimulus displays were static rather than continuous. Each stimulus comprised a given spatial extent (represented by its terminal points) and a given temporal extent (represented by a filled square displayed for a predetermined duration). Thus, each stimulus comprised the following sequence: Termini (2 dots) were presented on the screen, separated by a different constant distance, 1 s after the onset of which appeared the square (1 cm × 1 cm), which was displayed for a different constant duration. The terminal dots were presented along with the square and remained for an additional 1.5 s after the square went off. The termini always appeared in the lower part of the stimulus display; the square appeared in the upper part at a fixed centered position. In all other aspects conditions were nearly identical to Experiment 1.

Procedure. Stimuli comprised the same factorial combination of six values of distance and six values of duration used in Experiment 1. However, no motion was produced. As described before, the distance and duration components were presented separately, though concurrently, within the framework of a static display.

The subject was told to mentally construct a linear motion whose distance was defined by the magnitude of separation between the two dots (the left dot was described as the starting point), and whose duration was given by the temporal interval defined by the duration of the square stimulus. Subjects were instructed to judge the overall impression of the movement implied by the total stimulus event, not to assign numbers to the individual distance and duration components and divide the numbers.

Presented with a given stimulus display containing the necessary distance and duration information, the subject was told to estimate the velocity of the relevant inferred motion. Each subject was presented two replicates of the Distance × Duration matrix in the first session and three replicates in a second session (in a pseudorandom order). Thus, each subject made five judgments per stimulus in all. Again, the method was free magnitude estimation: The subject's task was to assign numbers in proportion to subjective velocity.

Results and Discussion

Pooled data. Figure 5 shows the principal results. The magnitude estimates of cognitive velocity given to each stimulus display were averaged geometrically, and these means are plotted on a linear axis as a function of log distance. A different curve is drawn for each duration.

The most salient characteristic of this family of curves is the roughly equal spacing in the vertical dimension. Neighboring pairs of functions appear to be separated from one another by a constant amount. The parallelism of the functions implies that the data conform to a subtractive structure operating on the psychological representations of distance and duration. Parallelism means that the amount of subjective velocity added (or subtracted) by any level of either component is independent of the level of the other component. Despite some variability, the overall form of the functions suggests that a subtractive model holds for the pooled data, at least to a first-order approximation. Analysis of variance, similar to that performed on the real velocity data, bears out the visual conclusion: The interaction term is nonsignificant, $F(25, 150) = 1.26, p > .05$.

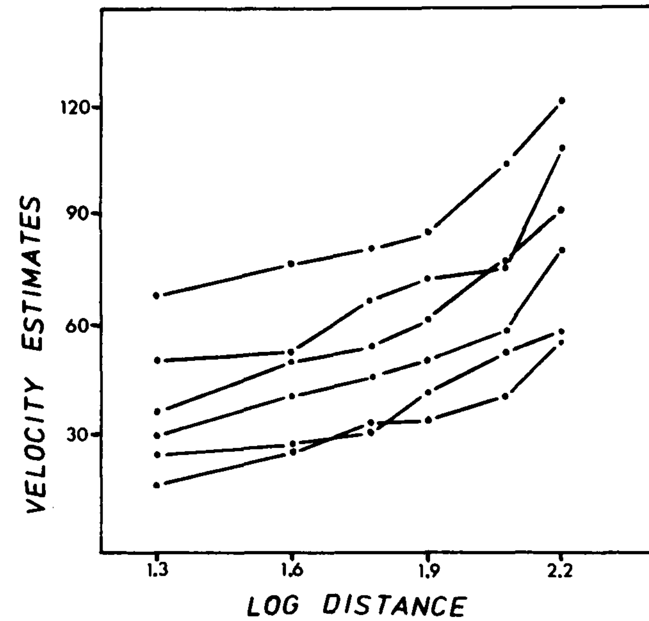

Figure 5. Magnitude estimates of cognitive velocity as a function of distance. (The parameter is duration.)

The present Velocity = Distance − Duration rule is striking, for it was obtained with adults under explicit instructions to estimate velocity while the necessary stimulus components were still being displayed. It sharply contrasts with the dividing structure extracted from the velocity judgments of Experiment 1 based on real movement. These data indicate that different metric structures underlie the processing of real velocity and the processing of cognitive velocity. The psychological values of the components relate to each other in different ways in the two paradigms. Scale values divide automatically in the perceptual system when subjects are presented with moving objects, but they assume a subtractive form of integration when velocity is evoked from a discontinuous display under the experimenter's direction.

The appearance of the subtracting Distance − Duration rule may seem a surprise in more than one respect. Stimuli in the cognitive velocity paradigm were derived from a factorial combination of the same values of the component physical variables as those serving in the real velocity experiment. Moreover, a two-dimensional representation of the physical stimulus is presumably much the same processing prerequisite in judging cognitive movement as in judging real movement. Observers clearly take account of both stimulus dimensions in cognitive velocity as they do in registering real movement (Experiment 1; Algom & Cohen-Raz, 1984). Yet beyond this basic similarity in the initial multiple representation of velocity, the two sets of internal scale values obey different algebraic rules of integration. Real motion is processed according to the normative Distance ÷ Duration rule that results almost automatically from a direct perceptual response to velocity. Motion imputed by subjects to a static display evidences, in contrast, an additive structure, and in particular, linear subtraction.

Curiously in this regard, Wilkening (1981) has found a subtractive rule of composition for velocity judgments of both children and adults. In Wilkening's developmental study, information about distance and duration was presented in two separate events (rather than intermixed into a single stimulus event). Given the existence of the experimental conditions for cognitive velocity (not his interpretation), the appearance of the subtractive structure should come as no surprise. Rather than indicating a developmental failure in understanding the relations among velocity, time, and distance so that "even the judgments of adults did not follow the correct integration model" in that "they fell back to a subtracting rule" (Wilkening, 1981, p. 241), the subtractive structure is the standard mode of subjective velocity production from discontinuous stimulus displays. In a strict sense, then, Wilkening did not test (real) velocity perception; what he examined was cognitive velocity.

Figures 6 and 7 give the psychophysical functions for cognitive velocity derived, respectively, for distance and duration from the appropriate marginal means. The fits to the power functions are excellent (r^2 equals .993 for the distance function and .955 for the duration function). The corrected exponent of the power function relating cognitive velocity to distance is 0.72 ($r^2 = .998$), and a parallel power fit applied to cognitive velocity as a function of (the reciprocal of) duration yields an exponent of 0.56 ($r^2 = .947$).

The scales of velocity underlying the perception of real movement (Experiment 1) and cognitive movement (this experiment) differ substantially. The cognitive scales contrast markedly with the respective perceptual scales, the former being consistently more compressive. In addition, cognitive velocity grows faster with increases in distance than with decreases in duration. The opposite, it should be recalled, was true for real velocity. Thus, besides mere alteration of scale values, the relative importance in determining subjective velocity assumed by the component physical variables changes in the two paradigms. Duration is the more influential variable in the perception of real movement, (Experiment 1; Algom & Cohen-Raz, 1984), whereas information about distance carries more weight in the determination of cognitive velocity (this experiment).[4]

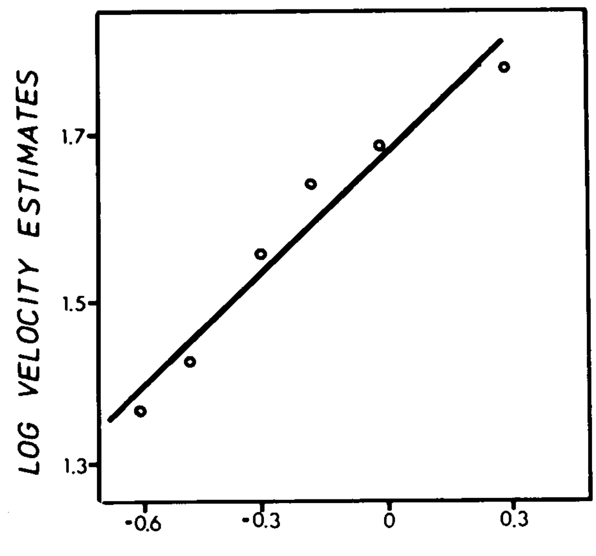

Figure 7. Marginal means of cognitive velocity judgments as a function of duration.

Figure 8 gives the overall psychophysical relation. Note that the graphic display depicts velocity estimates against physical velocities that were not actually presented but produced by dividing a static linear extent information by a static time information. The fit to the corrected power function is good ($r^2 = .920$); the exponent is 0.62. This value is considerably smaller than 0.85, the magnitude of the comparable exponent based on the real movement estimates in Experiment 1.

Individual data. Visual inspection of each subject's velocity contours—from graphical displays analogous to those shown in Figures 1 and 5—shows that despite greater noise in the individual data, in almost all cases the functions are about equally displaced in the vertical plane. Analyses of variance confirmed that the data for 6 of the 7 subjects (including the two subjects still available from the previous pool) had nonsignificant interaction terms, implying subtractivity of their numerical responses. For these subjects, $F(25, 100) < 1.18, p > .05$. For the remaining subject the interaction term was significant, $F(25, 100) = 1.84, p < .01$.

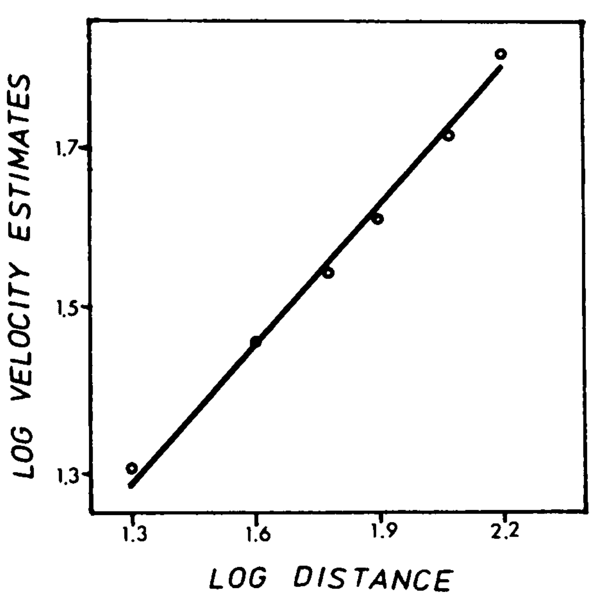

Figure 6. Marginal means of cognitive velocity judgments as a function of distance.

[4] Dominance of the duration component in determining the velocity of moving targets (see also Lappin et al., 1975; Rosenbaum, 1975) is probably related to the often made observation (e.g., Coren, Porac, & Ward, 1984, p. 321; Dembitz, 1927; Ellingstad & Heimstra, 1969; Gerhard, 1959; Johansson, 1950) that humans experience velocity as the time required for a target to travel some distance, rather than the distance traveled per unit of time. This temporal hypothesis of velocity is able to explain the consistently steeper velocity scales for duration obtained in this and in our previous investigation. Salient presentation of the characteristics of motion in a static display disrupts this perceptual predisposition, giving distance a prominent role.

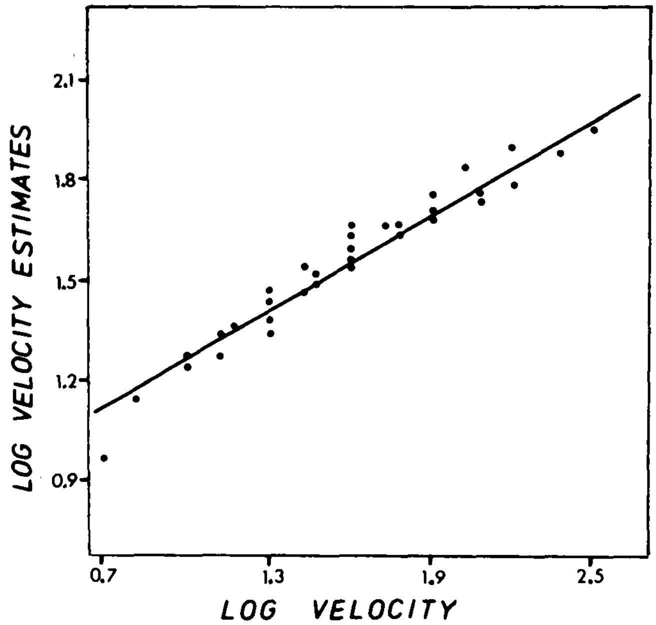

Figure 8. Psychophysical function for cognitive velocity: Magnitude estimates plotted against calculated physical velocity.

Perhaps the most remarkable demonstration of the evidence for multiple representations for subjective velocity comes from an explicit contrast of the individual data for subjects who took part in both experiments. Figure 9 gives the factorial plots for the data of one such subject (JB, No. 8 in Experiment 1, No. 6 in this experiment). It can be seen that the pattern of results is different in the two stimulus conditions. For the perceptual judgments of real movement in the left panel, the data follow the linear fan shape, implying a dividing, Distance ÷ Duration stimulus integration rule; however, the velocity estimates of the same subject undergo systematic changes when she is presented with the same distance and duration information appearing separately in a discontinuous display (right panel). The curves in the right panel are roughly parallel. This implies a subtracting, Distance − Duration integration rule. Analyses of variance performed on the two data sets of this subject confirmed the visual appearance of a dividing pattern for real velocity but a subtracting pattern for cognitive velocity. For real velocity, the analysis yielded a significant interaction term, $F(25, 100) = 1.81$, $p < .05$, with 37% of the interaction concentrated in the bilinear component (see Table 1); for cognitive velocity, the overall interaction was not significant, $F(25, 100) = 0.85$, $p > .05$. The other subject who participated in both experimental conditions (BB, labeled No. 2 in the real-movement condition and No. 3 in the cognitive condition) showed essentially the same results visually as well as statistically. Thus, the difference between the metric rules for real and cognitive velocity rests on the nature of psychological reality, not on artifacts of statistical grouping.

Table 3 presents the data on individual psychophysical functions. Columns 2–3, 4–5, and 6–7 give the slopes and r^2s of the corrected power functions of cognitive velocity, respectively, for physical velocity (recall, it was not presented as such), for distance, and for duration as independent variables.

The fits to the power functions are moderate. The most striking feature of the results, however, is their reasonable agreement with the markedly compressive cognitive velocity scales derived from the pooled data. That different sets of psychological values—different velocity scales—apply to real movement and to static displays is given, therefore, ample support in the individual results. The three sets of individual power function exponents for cognitive velocity (Table 3) clearly differ (the exponents are markedly smaller) from the parallel pool of exponents for real velocity (Table 2). The individual data display yet another characteristic of cognitive velocity that was apparent in the group results. For 4 of the 7 subjects, the exponent is greater for the distance function than for the duration function, which may imply a dominance of distance information over duration information in determining cognitive velocity.

In summary, the results indicate that when velocity judgments are associated with motionless stimulus displays, fundamental (metric and scale) properties of regular visual velocity estimates are altered. These results provide strong support for the conclusion that physical motion is a necessary condition for the appearance of the correct dividing stimulus integration rule. Degeneration of the velocity-inducing stimulus to the point of complete elimination of physical motion (in cognitive velocity judgments, in the kappa and tau effects, as well as in the large variety of situations that come under the label of apparent movement) causes a shift to a (more primitive?) subtractive structure operating on markedly more compressive scale values.

General Discussion

Different Integration Models

Two different rules for stimulus integration have appeared in the present data. Each deserves consideration. The results from the real movement condition are consistent with the hypothesis of a ratio model operating in the integration of distance and duration information onto subjective velocity. The finding of a clear Velocity = Distance ÷ Duration perceptual composition rule for the registration of real movement replicates the results of our previous study (Algom & Cohen-Raz, 1984). This dividing model contrasts sharply with a subtractive rule of integration found for the judgments extracted from the discontinuous stimulus displays in the cognitive velocity condition. This rule is surprising because the physical structure of the stimulus still demands a dividing mode of integration. Had subjects employed explicit mental calculations (e.g., Butler & Overshiner, 1983) or followed the logical operations implied by the required velocity judgment, a ratio model would have appeared for the cognitive velocity judgments as well. The failure of the normative Distance ÷ Duration model to reappear in estimates of implied (as opposed to real) motion supports the notion of an independent system operating in the processing of cognitive velocity as well as in other instances of imputation of movement to static environments (e.g., the kappa and tau effects, Helson & King, 1931; Huang & Jones, 1982; Jones & Huang, 1982). This latter velocity system acts in accord with an inherently noninteractive, Velocity = Distance − Duration rule of stimulus integration.

Not only do integration rules differ in the two paradigms, but

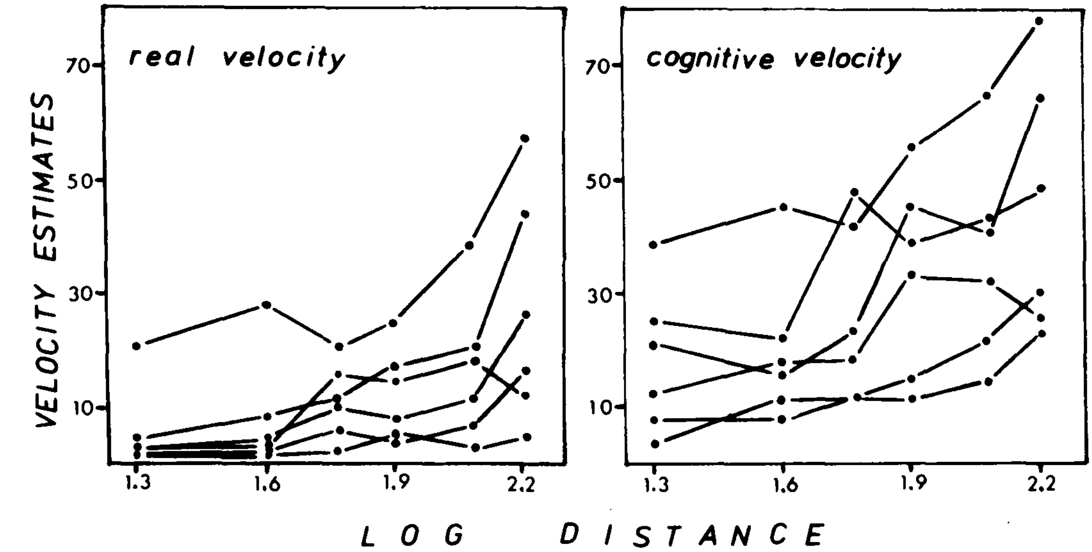

Figure 9. Magnitude estimates of real velocity and cognitive velocity, as in Figures 1 and 5, for 1 subject.

they probably rest on different levels of psychological processing. We concur with Lappin, Bell, Harm, and Kottas (1975) and Rosenbaum (1975), who maintain that the velocity of moving targets is perceived directly as a primary visual attribute (rather than indirectly by noting the changes in both spatial and temporal positions of objects). However, by no means should this "directness" of real velocity judgments be taken to indicate a failure on the part of the observer's sensory system to recover both distance and duration information. On the contrary, information from both dimensions is fully retained, though the components combining in the processing of real velocity are not directly available to perception (see Algom & Cohen-Raz, 1984). In contrast, the integration of separately presented spatial and temporal information into subjective velocity is wholly cognitive (rather than sensory or perceptual). In fact, integration in this case is optional, for the subject sees no movement. It is induced by directing the subject to mentally construct the appropriate motion and then to judge its velocity. The difference in processing between the two experimental conditions might profitably be put in terms of the operative cognitive unit.

Table 3
Power Function Exponents (Slopes) and r^2s Derived From the Marginal Means of the Factorial Designs for 7 Subjects

	Velocity		Distance		Duration	
Subject	Slope	r^2	Slope	r^2	Slope	r^2
1	0.883	.742	0.857	.996	0.522	.913
2	0.375	.740	0.446	.956	0.386	.952
3	0.661	.691	0.799	.967	0.269	.859
4	0.442	.857	0.365	.995	0.496	.929
5	0.531	.862	0.618	.971	0.828	.985
6	1.715	.805	1.716	.984	1.807	.890
7	0.492	.819	0.868	.987	0.845	.924
M	0.586		0.667		0.593	

In the perception of real movement it is the very movement or its velocity that is the cognitive unit. When distance and duration are presented separately, these dimensions form the cognitive unit (to be integrated in the case of velocity judgments). Following the distinction urged by Marks (1979a) in his related study, the integration taking place in the processing of real movement can be designated as *synthetic* integration, whereas the one operating in cognitive velocity can be termed as *analytic* integration.

Finally, the limitations of this study in drawing general conclusions about perceptual and cognitive algorithms for velocity should be clearly appreciated. It should be recognized that both experiments in this study used simple stimulus displays of only one or two components in uncluttered frameworks. We do not know whether the same quantitative relations hold for motion in other environments containing multiple moving and stationary objects of varying sizes and positioned at varying distances from the target (cf. J. F. Brown, 1931c, Epstein & Cody, 1980; see also Epstein, 1978). Indeed, even the dimensionality (D)—2D or 3D—for describing metric relations involved in the perceived spatial, temporal, and velocity variations strongly depends on the positions and motions of neighboring components. Clearly, specification of the metric structure(s) and psychophysical relations that underlie the processing of velocity in articulated spaces of differing perceived dimensionality has yet to be accomplished.

Some Implications for Velocity Research

The experimental condition designated *cognitive velocity* is based on a unique set of stimulus and response properties. It shares some properties with real motion and other properties, probably the more significant ones, with apparent motion and movement-related illusions. Table 4 summarizes some of the similarities and differences. A striking feature of the classificatory scheme summarized in Table 4 is the appearance of an underlying similarity between the present cognitive velocity

Table 4
Experimental Conditions and Results of Studies of Movement and Movement-Related Psychological Phenomena

Condition	Stimulus display	Appearance of motion?	Response related to motion?	Metric structure	Psychophysical relation
Motion					
Real	Continuous	Yes	Yes	Divisive	Mod. comp.
Cognitive	Stationary	No	Yes	Additive	Str. comp
Apparent	Stationary	Yes	Yes[a]	Undetermined	Str. comp.
Concealed	Stationary[d]	No[e]	Yes[a]	Undetermined	Str. comp.
Kappa & tau effects	Stationary	?[b]	No	Additive[c]	Undet.

Note. Mod. comp. = moderately compressive; Str. comp. = strongly compressive; Undet. = undetermined.
[a] Sometimes other responses are required such as distance or time estimation or to indicate whether a collision between targets is imminent.
[b] No published data are available on this interesting question.
[c] In general, linear combination models apply.
[d] At least for the period that the target is not available to visual perception.
[e] Again, no motion is, of course, seen while the target is concealed from the observer.

condition and other movement-related illusions based on discontinuous displays. Consider first the psychophysical contingencies. Kolers (1964) has pointed out that the speed of an object in real movement has to be made less than the calculated speed of an object in illusory movement in order for their velocities to appear equal (see also Kolers, 1963, 1972). Similarly, the tendency of most observers to underestimate the rate of inferred displacement of concealed targets (stimuli displayed intermittently for view) is also consistent with the presently derived, markedly compressive functions for cognitive velocity (e.g., Ellingstad & Heimstra, 1969; Jones & Heimstra, 1964; Morin, Grant, & Nystrom, 1956). Both (a) the distinctively compressive psychophysical functions obtained for real velocity and (b) the fact that subjects underestimate velocity in both apparent and concealed motion *relative* to appropriate real motion standards argue for the existence of sharply compressive psychophysical relations in the latter conditions.

Fewer data are available on metric structure. Yet the only motion-related illusions studied in that respect—the kappa and tau effects (Helson & King, 1931; Cohen, Hansel, & Sylvester, 1953, 1955; Price-Williams, 1954)—display well-defined additive models similar to the one found here underlying cognitive motion. In sum, notwithstanding important subjective differences (e.g., the appearance of motion) that exist between cognitive motion and various apparent motion phenomena (and the need for more relevant data), we suggest the hypothesis that the presently uncovered features of cognitive motion (metric structure and psychophysical relations) apply in all cases of a velocity response evocation from discontinuous displays.

This has important consequences, including the need for possible revision of some widely accepted theoretical positions. The rather extensively scrutinized kappa and tau effects can serve to illustrate this point. They relate either to an effect of temporal extent on estimates of distance (tau effect) or an effect of spatial extent on estimates of time (kappa effect). What makes these illusions especially relevant to the present concerns is their widely accepted explanation in terms of a uniform motion (constant velocity) imputed by subjects to a discontinuous display. This is the so-called "constant velocity hypothesis" (e.g., Anderson, 1974; Collyer, 1977; Huang & Jones, 1982; Jones & Huang, 1982; Price-Williams, 1954). The present results and hypothesis call for a reconsideration of the constant velocity notion. Subjects may well impute motion to discontinuous fields, yet the motion they impute is cognitive motion obeying qualitatively different rules than do perceptions of real motion.

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